

Structure of Matter - I
April 6, 2014

PROBLEM 1. On electronic structure [15 ptn]

Consider an excited V atom ($1s^2 2s^2 2p^6 3s^2 2p^6 4s^2 3d^2 5d$).

- a) Calculate the angle between the angular momentum vector and the z axis for a single 5d electron with $m=-2$. [1 ptn]
- b) Sketch the radial part of this 5d wave function. [1 ptn]
- c) Calculate the binding energy of H(5d). [1 ptn]
- d) Explain why this excited V (.....5d) configuration is equally strong, stronger or weaker bound than a 5d electron in H, i.e., H(5d). [2 ptn]
- e) After some time the V atom has decayed to its ground electronic configuration $1s^2 2s^2 2p^6 3s^2 2p^6 4s^2 3d^3$. What is the most likely decay pathway? [2 ptn]

Now consider V in its ground electronic configuration $1s^2 2s^2 2p^6 3s^2 2p^6 4s^2 3d^3$.

- f) Determine the ground term of V. [3 ptn]
- g) Determine the ground level of V. [1 ptn]
- h) The V atom has a nuclear spin of $I=7/2$. Due to the nuclear spin the ground level splits up into hyperfine levels. Determine all the possible hyperfine levels. (In case you could not determine the ground level (question 1g) then you may use $J=5/2$). [1 ptn]
- i) What are the answers to questions 1f), 1g), and 1h) when V is replaced by Co which has not 3 but 7 electron in the 3d subshell? Co has $I=7/2$ just like V. [3 ptn]

PROBLEM 2. On nuclear structure [10 ptn]

Consider the vanadium nucleus $\begin{matrix} 51 \\ 23 \end{matrix} \text{V}$.

- a) Calculate the charge density [in units of elementary charge per fm^3] of the isotope. [1 ptn]
- b) Determine the nuclear spin J and the parity of this isotope. [4 ptn]
- c) Determine the lowest excited nuclear level? [2 ptn]
- d) For nuclei the binding energy per nucleon can well be described by the semiempirical formula. Derive that for all nuclei with 51 nucleons, vanadium has the highest binding energy per nucleon. [3 ptn]

PROBLEM 3. Elementary particles [10 ptn]

Consider a charmed Λ_c baryon with quark content udc which decays into $p + K^- + \pi^+$.

- Determine the charge of this Λ_c baryon? [1 ptn]
- Determine the hypercharge of this Λ_c baryon? [1 ptn]
- Verify that the conservation laws for lepton and baryon number are respected? [1 ptn]
- Which conservation law is violated? [1 ptn]
- By which force(s) is the decay driven? [1 ptn]
- What is the approximate time scale of the decay? [1 ptn]
- Consider the $J=3/2$ family of charmed baryons. The 6 baryons of this family are: udc , usc , dsc , uuc , ddc , and ssc . Determine the hypercharge Y , the azimuthal isospin I_3 , and the isospin of these six baryons. [2 ptn]
- What are the values of the color charges I_3^C and Y^C of the udc baryon? [2 ptn]

Helpful information:

$$B(N, Z) = aA - bA^{2/3} - \frac{dZ^2}{A^{1/3}} - s \frac{(N-Z)^2}{A} - \frac{\delta}{A^{1/2}}$$

$$\begin{aligned} a &= 15.84 \text{ MeV} \\ b &= 18.33 \text{ MeV} \\ d &= 0.714 \text{ MeV} \\ s &= 23.20 \text{ MeV} \end{aligned}$$

$$\delta = \begin{cases} +11.2 \text{ MeV} & \text{odd-odd} \\ 0 & \text{ev.-odd} \\ -11.2 & \text{ev.-ev.} \end{cases}$$

$$K^- : s\bar{u}$$

$$\pi^+ : u\bar{d}$$

$$R_{nuc} = 1.12 A^{1/3} \text{ fm}$$

sequence of nuclear shell filling.
1s, 1p, 1d, 2s, 1f, 2p, 1g, ...